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INTERFERENCE FRAGMENTATION FUNCTIONS IN DEEP INELASTIC SCATTERING

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We give a general overview of the rôle and interpretation of fragmentation functions in hard processes. Transverse momentum dependence gives rise to time-reversal odd fragmentation functions contributing at leading order. Final state interactions are necessary to have non-zero time reversal odd functions. We present a model calculation for the special case of interference fragmentation functions in two-hadron inclusive lepton hadron scattering.

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1 Physics motivation

Hard processes like deep-inelastic lepton-hadron scattering (DIS), the electron/positron annihilation (e^+e^-), or lepton-pair poduction in hadron-hadron scattering probe the *internal structure of hadrons in terms of quark and gluon degrees of freedom.*

The information obtained on the hadronic structure is encoded in *distribution functions* (DF) and *fragmentation functions* (FF). Those functions taken from one experiment can be used without change to predict the results of other hard processes. The property of independence from the particular reaction, the so-called *universality*, is assured by the existence of factorization theorems for some of the hard processes[1], whereas for others factorization is used as a plausible assumption.

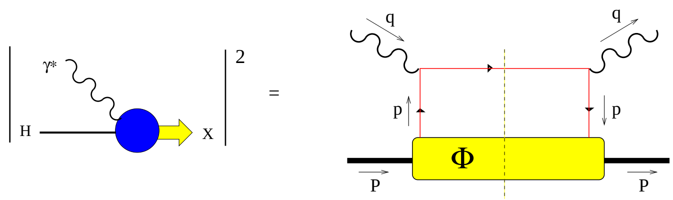
Leading twist distribution and fragmentation functions (i.e. those which contribute to leading order in an expansion of inverse powers of the hard scale Q in the process under consideration) have a simple *probabilistic interpretation* in the context of lightcone quantization.

The parton model provides us with simple relations between DF/FF and the physical observables of the processes, like *cross sections* or *structure functions.*

2 Quark-quark correlation functions distribution/fragmentation functions

We illustrate the formal definitions of distribution/fragmentation functions and their relations to observables as given by the (naïve, i.e. not QCD corrected) parton model with two examples:

- The hadron tensor for *inclusive deep inelastic lepton-hadron scattering* is calculated in leading order from the "handbag" diagram



The famous parton model relation between the structure function F_1 (and F_2 via the Callan-Gross relation) and the DF f_1 is

$$2F_1(x_B) = \frac{F_2(x_B)}{x_B} = \sum_a e_a^2\, f_1^a(x_B) \tag{1}$$

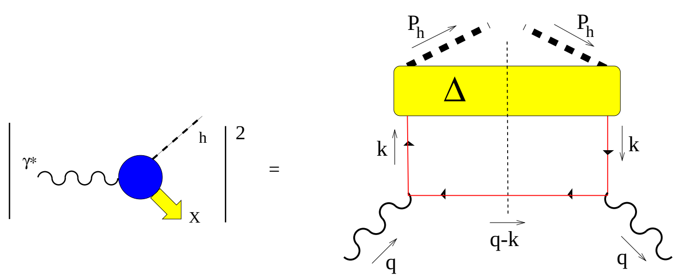
with $x_B = Q^2/(2P \cdot q)$ and summation over flavors a. A formal definition[2,3] of f_1 is obtained from a quark-quark correlation function

$$\Phi_{ij}(p;P,S) = \int \frac{d^4x}{(2\pi)^4}\, e^{ip\cdot x}\, \langle P,S|\overline{\psi}_j(0)\, \psi_i(x)|P,S\rangle \tag{2}$$

as (partially integrated) Dirac projection in the form

$$f_1(x) = \frac{1}{2}\int dp^-\, d^2\vec{p}_T\ \mathrm{Tr}(\Phi\ \gamma^+)\Big|_{p^+=xP^+} \tag{3}$$

- The leading order diagram for *one-hadron inclusive electron/positron annihilation* is



The parton model relation between the cross section, and the FF D_1 reads

$$\frac{d\sigma(e^+e^-)}{d\Omega dz_h} \sim \sum_a e_a^2\, D_1^a(z_h) \tag{4}$$

with $z_h = 2P_h \cdot q/Q^2$. A formal definition[2,3] of the FF D_1 is obtained from the quark-quark correlation function

$$\Delta_{ij}(k, P_h, S_h) = \sum\!\!\!\!\!\!\int_X \int \frac{d^4x}{(2\pi)^4}\, e^{ik\cdot x}\, \langle 0|\psi_i(x)|P_h, S_h; X\rangle\langle P_h, S_h; X|\overline{\psi}_j(0)|0\rangle \tag{5}$$

again as (partially integrated) Dirac projection

$$D_1(z) = \frac{1}{4z}\int dk^+\, d^2\vec{k}_T\ \mathrm{Tr}(\Delta\gamma^-)\Big|_{k^- = P_h^-/z}\ . \tag{6}$$

2.1 DF: partons in a hadron

With the definition

$$\Phi^{[\Gamma]}(x) = \frac{1}{2}\int dp^-\, d^2\vec{p}_T\ \mathrm{Tr}\,(\Phi\ \Gamma)\Big|_{p^+ = xP^+} \tag{7}$$

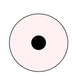
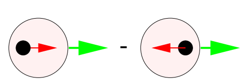
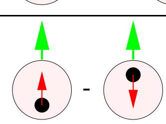
we can list all (x dependent) leading twist distribution functions as

$$\Phi^{[\gamma^+]}(x) = f_1(x) \qquad \text{momentum distribution} \quad [\text{also called: } q(x)] \tag{8}$$

$$\Phi^{[\gamma^+\gamma_5]}(x) = \lambda\, g_1(x) \qquad \text{helicity distribution} \quad [\Delta q(x)] \tag{9}$$

$$\Phi^{[i\sigma^{\alpha+}\gamma_5]}(x) = S_T^\alpha\ h_1(x) \qquad \text{transv. spin distr.}^4 \quad [\delta q(x), \Delta_T q(x)] \tag{10}$$

with λ being the helicity and S_T^α the transverse component of the spin vector. Within the framework of lightcone quantization[5] the leading twist distribution functions acquire a simple probabilistic interpretation:

interpretation:	quark helicity content:
$f_1 =$ 	$\overline{\psi}\,\gamma^+\,\psi = \sqrt{2}\,\psi_+^\dagger\,(P_R P_R + P_L P_L)\ \psi_+ = \overline{R}R + \overline{L}L$
$g_1 =$ 	$\overline{\psi}\,\gamma^+\gamma_5\,\psi = \sqrt{2}\,\psi_+^\dagger\,(P_R P_R - P_L P_L)\,\psi_+ = \overline{R}R - \overline{L}L$
$h_1 =$ 	$\overline{\psi}\,\mathrm{i}\sigma^{i+}\gamma_5\,\psi = \sqrt{2}\,\psi_+^\dagger\ \left(P_L\gamma^i P_R - P_R\gamma^i P_L\right)\psi_+ = \overline{L}R - \overline{R}L$

The DF $f_1(x)$ and $g_1(x)$ are known rather well from experiment. The chiral odd DF $h_1(x)$ presently is completely unknown. An obstacle to its determination is the fact that it needs another chiral odd function to be combined with in an observable.

2.2 FF: hadrons in a parton

Information on the hadronic structure which is complementary to the one encoded in the distribution functions is contained in *fragmentation functions*, which describe the hadronization process of a (current) quark to hadrons. With the general definition

$$\Delta^{[\Gamma]}(z) = \frac{1}{4z} \int dk^+ \, d^2\vec{k}_T \; \mathrm{Tr}(\Delta\,\Gamma)\Big|_{k^- = P_h^-/z} \tag{11}$$

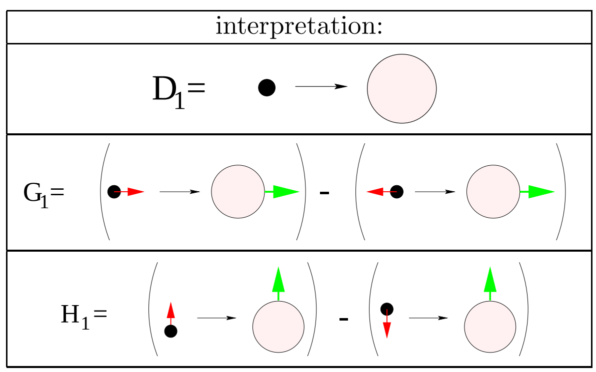
we can list all (z dependent) leading twist fragmentation functions as

$$\Delta^{[\gamma^-]}(z) = D_1(z) \tag{12}$$

$$\Delta^{[\gamma^-\gamma_5]}(z) = \lambda_h \, G_1(z) \tag{13}$$

$$\Delta^{[i\sigma^{\alpha-}\gamma_5]}(z) = S_{hT}^{\alpha} \, H_1(z) \tag{14}$$

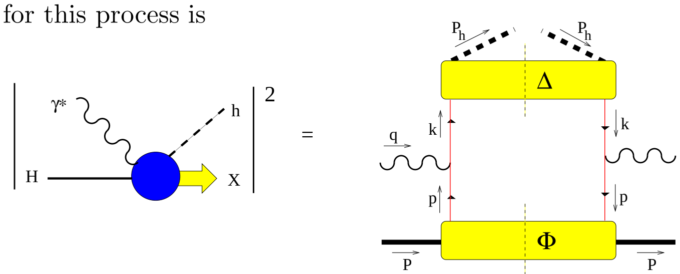
which have a probabilistic interpretation in analogy to the one of DF. The only FF known from experiment is $D_1(x)$ for some species of hadrons, like protons, neutrons, pions and kaons.



2.3 Transverse momentum dependent FF

There are hard processes in which information on the transverse momenta of quarks relative to their parent hadrons is retained if the observables are kept differential in the transverse momentum of one of the external momenta. This leads to a larger number of independent DF and FF.[6]

Consider, for instance, the semi-inclusive DIS process: $\ell + H \to \ell' + h + X$, where one of the hadrons in the final state is observed. The leading quark diagram for this process is

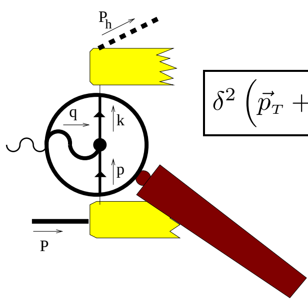


The three external momenta in general are <u>not</u> collinear; at least one of them will have a transverse component. A possible parametrization in a frame where target momentum and momentum of the produced hadron are collinear is (in lightcone coordinates $a^\mu = [a^-, a^+, \vec{a}_T]$)

$$P = \left[\frac{x_B M^2}{Q\sqrt{2}}\,,\;\frac{Q}{x_B\sqrt{2}}\,,\;\vec{0}_T\right],\quad P_h = \left[\frac{z_h Q}{\sqrt{2}}\,,\;\frac{M_h^2}{z_h Q\sqrt{2}}\,,\;\vec{0}_T\right]\;.$$

TIn this frame the photon momentum has a transverse component

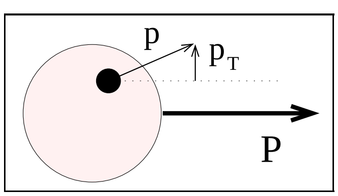
$$q = \left[\frac{Q}{\sqrt{2}}\,,\;-\frac{Q}{\sqrt{2}}\,,\;\vec{q}_T\right]\;.$$

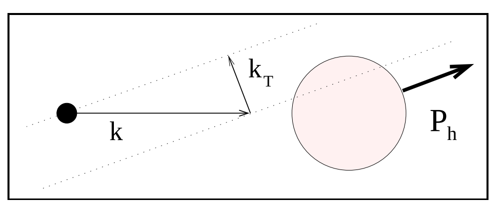


$$\boxed{\delta^2\left(\vec{p}_T + \vec{q}_T - \vec{k}_T\right)}$$

The transverse component of the photon momentum $\vec{q}_T$ is connected to the quark transverse momenta by the momentum conservation of an elementary vertex in the hard part of the quark diagram. Thus, the cross section differential in the photon transverse momentum $d\sigma/(d\ldots d^2\vec{q}_T)$ is sensitive to the quark transverse momenta $\vec{p}_T$ and $\vec{k}_T$ relative to their parent hadrons.

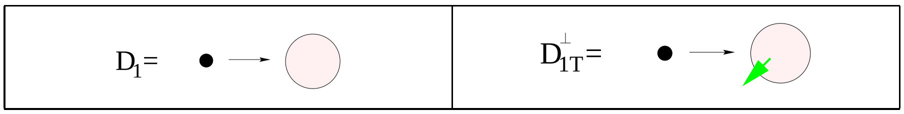
Quark transverse momenta relative to hadrons in DF and FF, respectively:



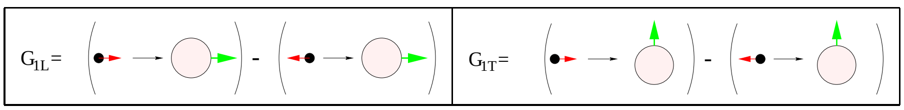


The leading order (z_h and $\vec{k}_T$ dependent) FF are:

$$\Delta^{[\gamma^-]}(z_h, \vec{k}_T) = D_1(z_h, \vec{k}_T^{\,2}) + \frac{\epsilon_{Tij} k_T^i S_{hT}^j}{M_h} D_{1T}^{\perp}(z_h, \vec{k}_T^{\,2}) \tag{15}$$

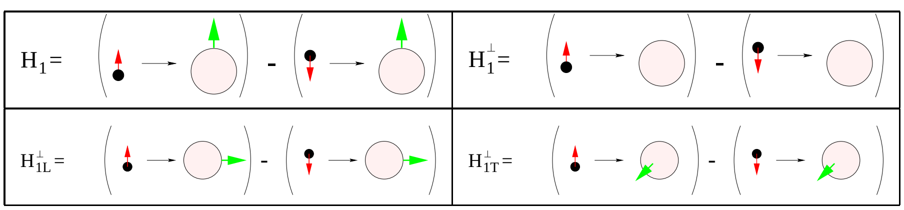


$$\Delta^{[\gamma^-\gamma_5]}(z_h, \vec{k}_T) = \lambda_h G_{1L}(z_h, \vec{k}_T^{\,2}) + \frac{\vec{k}_T \cdot \vec{S}_{hT}}{M_h} G_{1T}(z_h, \vec{k}_T^{\,2}) \tag{16}$$



$$\Delta^{[\mathrm{i}\sigma^{i-}\gamma_5]}(z_h, \vec{k}_T) = S_{hT}^i H_1(z_h, \vec{k}_T^{\,2}) + \frac{\epsilon_T^{ij} k_{Tj}}{M_h} H_1^{\perp}(z_h, \vec{k}_T^{\,2})$$

$$+ \frac{\lambda_h k_T^i}{M_h} H_{1L}^{\perp}(z_h, \vec{k}_T^{\,2}) + \frac{\left(k_T^i k_T^j - \vec{k}_T^{\,2}\delta_{ij}/2\right) S_{hT}^j}{M_h^2} H_{1T}^{\perp}(z_h, \vec{k}_T^{\,2}) \tag{17}$$



The FF $D_{1T}^{\perp}$ and $H_1^{\perp}$ are time reversal odd (T-odd). Here the notion "time reversal odd" is used in the sense: *"in the absence of final state interactions (FSI) those functions would be forbidden by a constraint from time reversal invariance"*.

The interest in T-odd fragmentation functions was triggered by a proposal to measure the transversity distribution h_1 from an azimuthal asymmetry in one-hadron inclusive DIS which involves $H_1^{\perp}$ as the necessary chiral odd partner [7]

$$\frac{d\sigma}{d\ldots d^2\vec{q}_T} \sim \sin(\phi)\ h_1(x, \vec{p}_T^{\,2})\ H_1^{\perp}(z_h, \vec{k}_T^{\,2})\ . \tag{18}$$

3 Two-hadron fragmentation functions

3.1 Why two-hadron FF ?

The modeling of T-odd one-hadron FF with specific assumptions on the hadronizatin process encounters serious difficulties. A necessary ingredient is the existence of at least two competing channels interfering through a non-vanishing phase. Moreover, it turns out that a genuine difference in the Lorentz structure of the vertices describing the fragmentation is needed.

On general grounds, model assumptions for an interaction of the observed hadron with the rest of the jet have either problems with factorization breaking, with translational and rotational invariance, or they can be redefined in the quark/hadron/rest-of-jet vertex, thus not resulting in the necessary modification of the Lorentz structure. Moreover, it was also argued[8] that the required sum over all possible states of the fragments could average out the FSI effects.

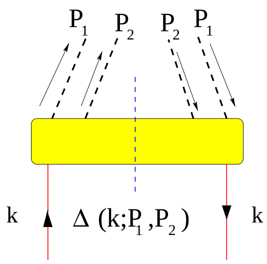
These arguments naturally lead to the consideration of final state interactions between two hadrons emitted inside the same jet as a source for T-odd fragmentation functions.

For the case of the two hadrons being a pair of pions the resulting FF have been proposed to investigate the transverse spin dependence of fragmentation. Collins and Ladinsky[9] considered the interference of a scalar resonance with the channel of independent successive two pion production. Jaffe, Jin and Tang[8] proposed the interference of s- and p-wave production channels, where the relevant phase shifts are essentially known.

3.2 Definition of two-hadron FF and symmetry properties

Consider the situation where a pair of hadrons is produced in the same quark current jet: $lH \to l'h_1h_2X$. By generalizing the Collins-Soper light-cone formalism[2] for fragmentation into multiple hadrons the corresponding quark-quark correlation function can be defined

$$\Delta(k;P_1,P_2) = \sum\!\!\!\!\!\!\int_X \int \frac{d^4\zeta}{(2\pi)^4}\, e^{ik\cdot\zeta}\, \langle 0|\psi_i(\zeta)|h_1,h_2,X\rangle\langle X,h_1,h_2|\overline{\psi}_j(0)|0\rangle \quad (19)$$



For the corresponding quark-quark correlation function $\Delta(k;P_1,P_2)$ one can make the most general ansatz (allowed by *parity invariance*)

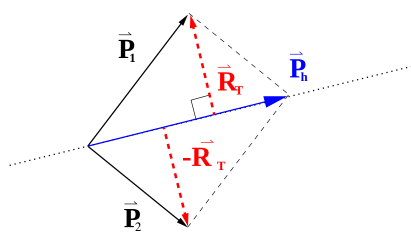
$$
\begin{aligned}
&\Delta(k;P_1,P_2) = \\
&\quad C_1\ (M_1+M_2) + C_2\ \not{P}_1 + C_3\ \not{P}_2 + C_4\ \not{k} \\
&\quad + \frac{C_5}{M_1}\,\sigma^{\mu\nu}P_{1\mu}k_\nu + \frac{C_6}{M_2}\,\sigma^{\mu\nu}P_{2\mu}k_\nu \\
&\quad + \frac{C_7}{M_1+M_2}\,\sigma^{\mu\nu}P_{1\mu}P_{2\nu} \\
&\quad + \frac{C_8}{M_1M_2}\,\gamma_5\,\epsilon^{\mu\nu\rho\sigma}\gamma_\mu P_{1\nu}P_{2\rho}k_\sigma\ . \qquad (20)
\end{aligned}
$$

From *hermiticity* follows $C_i^* = C_i$ for $i = 1-4, 5-8$, and a constraint from *time-reversal invariance* (if applicable) leads to $C_i^* = C_i$ for $i = 1-4$ and $C_i^* = -C_i$ for $i = 5-8$. FSI, however, render the constraint inapplicable, which otherwise would require $C_5..C_8 = 0$. FF involving $C_5..C_8$ are called T-odd.

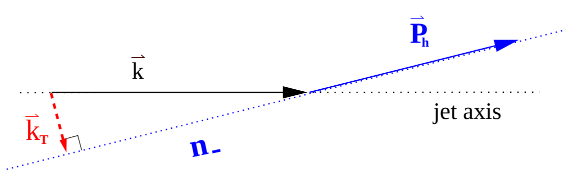
Before defining the two-hadron FF some kinematic quantities have to be introduced:

- sum of four-momenta of the hadron pair: $P_h = P_1 + P_2$
- momentum fractions:
  $P_1^- = \xi\ z_h\ k^-$
  $P_2^- = (1-\xi)\ z_h\ k^-$

- (half of) relative transverse momentum inside the hadron pair:
  $\vec{R}_T = (\vec{P}_{1T} - \vec{P}_{2T})/2$



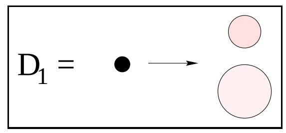
- relative transverse momentum between quark and hadron pair: $\vec{k}_T$



The two-hadron FF are defined as projections of $\Delta(k; P_1, P_2)$ on different Dirac structures. They depend on five variables: on the momentum fraction of the hadron pair (z_h), on the way this momentum is shared inside the pair (ξ), on the invariant mass of the pair M_h^2, and on the "geometry" of the pair, namely on the relative orientation between the hadron pair plane and the quark jet axis ($\vec{k}_T^{\,2}$, $\vec{k}_T \cdot \vec{R}_T$).

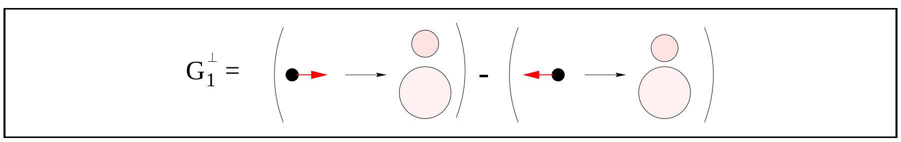
To leading order there are

$$\Delta^{[\gamma^-]}(z_h, \xi, \vec{k}_T^{\,2}, \vec{k}_T \cdot \vec{R}_T, M_h^2) = D_1 \qquad (21)$$



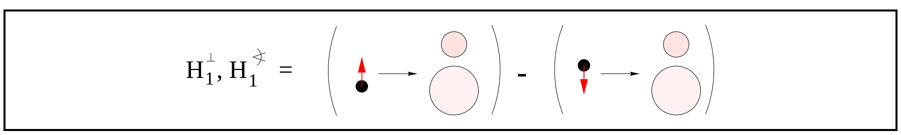
and the interference FF

$$\Delta^{[\gamma^- \gamma_5]}(z_h, \xi, \vec{k}_T^{\,2}, \vec{k}_T \cdot \vec{R}_T, M_h^2) = \frac{\epsilon_T^{ij}\, R_{Ti}\, k_{Tj}}{M_p M_\pi}\, G_1^\perp \qquad (22)$$



and

$$\Delta^{[\mathrm{i}\sigma^{i-}\gamma_5]}(z_h, \xi, \vec{k}_T^{\,2}, \vec{k}_T \cdot \vec{R}_T, M_h^2) = \frac{\epsilon_T^{ij}\, R_{Tj}}{M_p + M_\pi}\, H_1^{\sphericalangle} + \frac{\epsilon_T^{ij}\, k_{Tj}}{M_p + M_\pi}\, H_1^\perp \qquad (23)$$



Inserting the ansatz (20) for $\Delta(k; P_1, P_2)$ reveals that $G_1^\perp$, $H_1^{\sphericalangle}$ and $H_1^\perp$ are T-odd. From the Dirac matrices in the projections it can be deduced that D_1 and $G_1^\perp$ are chiral even, and $H_1^{\sphericalangle}$, $H_1^\perp$ are chiral odd.

3.3 Pion-Proton interference FF

Recently, we have calculated explicitely the two-hadron interference fragmentation functions for the case of the hadron pair being a pion and a proton (produced in the same jet)[10]. The calculation is done in an extended version

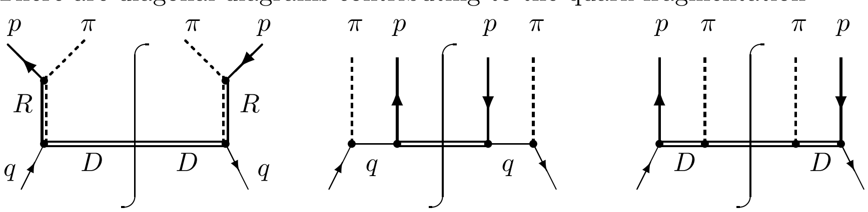
of the spectator model used in Ref. [11]. The interference of two channels is considered, where the hadron pair is produced either through a Roper resonance, or as successive independent production.

The basic idea of the spectator model is to make a specific ansatz for the spectral decomposition of the quark correlator by replacing the sum over the complete set of intermediate states in (20) with an effective spectator state with a definite mass M_D and the quantum numbers of the diquark.

As a consequence of the quark-quark correlation function simplifies to a form

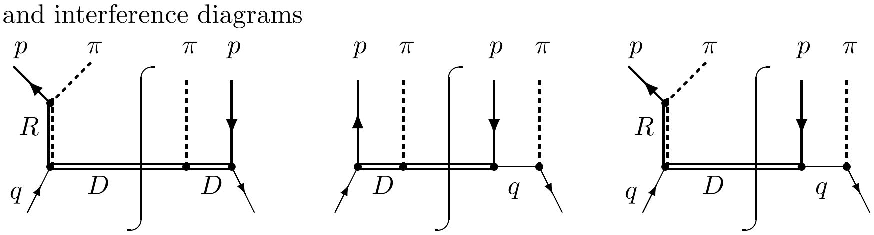
$$\Delta_{ij}(k;P_p,P_\pi) = \frac{\theta((k-P_h)^+)}{(2\pi)^3}\,\delta\left((k-P_h)^2 - M_D^2\right) \times\langle 0|\psi_i(0)|\pi,p,D\rangle\langle D,p,\pi|\overline{\psi}_j(0)|0\rangle \qquad (24)$$

containing amplitudes which can be calculated from Feynman diagrams. There are diagonal diagrams contributing to the quark fragmentation



$D-a$ $D-b$ $D-c$

and interference diagrams



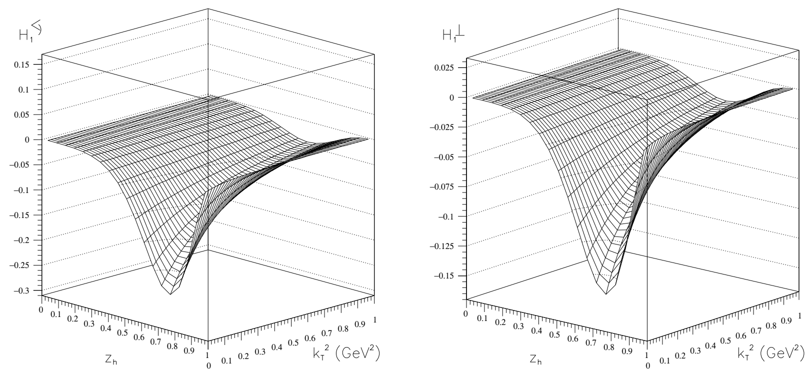
$I-a$ $I-b$ $I-c$

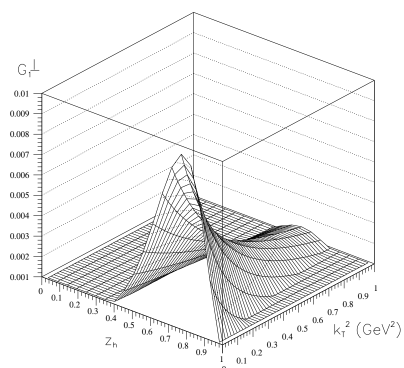
On the Roper resonance diagram D-a dominates the D_1 function, and I-a, I-c give the dominant contributions to the interference functions $G_1^\perp$, $H_1^\sphericalangle$, $H_1^\perp$.

For the explicit calculation all propagators and vertices occurring in the diagrams have to be specified. Particularly important are form factors at the vertices, which prevent the propagators from having large virtualities. The asymptotic behavior of the form factors has been deduced from quark counting rules, the strength of the couplings can be determined from phenomenological considerations. Details of this model calculation will be published elsewhere.[10]

4 Numerical results

For plotting the results we assume that the proton-pion pair has an invariant mass equal to the Roper resonance $M_h = M_R = 1.44$ GeV, and a special kinematic situation is chosen, where $\vec{k}_T \cdot \vec{R}_T = 0$ holds. Then the dependences reduce to $G_1^\perp(z_h, \xi, \vec{k}_T^2)$, etc.





The FF $H_1^{\sphericalangle\, u\to p+\pi}$, $H_1^{\perp\, u\to p+\pi}$, $G_1^{\perp\, u\to p+\pi}$ are shown as functions of z_h, $\vec{k}_T^2$ at a fixed value $\xi = 0.7$. Similar results are obtained for different values of ξ in the kinematically allowed range. The maximum sensitivity to the fragmentation mechanism is concentrated around the kinematical range where the pair takes roughly 80% of the jet energy ($z_h \sim 0.8$) and has a small transverse momentum with respect to the jet axis ($\vec{k}_T^2 \lesssim 0.4$ GeV^2).

Acknowledgments

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